A simple method to estimate tensile strength and Hoek-Brown strength parameter m_i of brittle rocks

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ABSTRACT: A practical simple method is proposed to estimate the tensile strength (σ_t) of strong brittle rocks from the strength ratio of $R= \sigma_c/\sigma_t = 8\ \sigma_c/\sigma_{ci}$, where σ_c is the uniaxial compressive strength and σ_{ci} is the crack initiation stress in an uniaxial compression test. 8 is the Griffith strength ratio and the term σ_c/σ_{ci} accounts for the difference of crack growth or propagation in tension and compression in uniaxial compression tests. σ_{ci} can be reliably obtained from volumetric strain measurement or acoustic emission (AE) monitoring. With the strength ratio R determined, the tensile strength can be indirectly obtained from the result of uniaxial compression tests. In addition, a practical estimate of the Hoek-Brown strength parameter m_i is presented and it is suggested that m_i = 12 σ_c/σ_{ci} can be applied to strong, brittle rocks in high confinement zone and m_i = 8 σ_c/σ_{ci} can be applied to low confinement to tension zones. It is found that the predicted tensile strengths and Hoek-Brown strength parameter m_i using this method are in good agreement with test data. The rock strength parameters like σ_t and m_i, which require specialty tests such as direct tensile (or Brazilian) and triaxial compression tests for their determination, can be reasonably estimated from uniaxial compression tests.

1 INTRODUCTION

1.1 *Rock tensile strength*

The tensile strength is generally low compared to its compressive strength. Direct tensile strength tests of rocks are not routinely conducted because of the difficulty in specimen preparation. Indirect methods, such as bending and Brazilian tests, are often used to obtain the tensile strength of rocks.

When tensile strength test data are not available, the general approach to estimate rock tensile strength makes use of the correlation between uniaxial compressive strength (σ_c) and tensile strength (σ_t) and applies the generally agreed relationship of $\sigma_c = R \cdot |\sigma_t|$, where $R \approx 10$ (Sheorey 1997). Estimation of rock tensile strength using $|\sigma_t| = \sigma_c /10$ is only a first step but definitely not the best approach. The test dataset compiled by Sheorey (1997), although limited in number, shows a large variation of the strength ratio ($R = \sigma_c/|\sigma_t|$), from 2.7 to 39 with an average of 14.7. Vutukuri et al. (1974) stated that the strength ratio of most rocks varies from 10 to 50. Brook (1993) noticed that the strength ratio R depends on rock type. However, the strength ratios of sandstone compiled by Sheorey (1997) range from 7 to 39 with an average of 14.9. With such a large range of data variation and lack of direct correlation of tensile strength with rock type, it presents a significant challenge for engineers who want to infer a strength ratio from the database.

1.2 *Hoek-Brown strength parameter m_i*

The Hoek-Brown failure criterion (Hoek & Brown 1980a, Hoek & Brown 1997, Hoek et al. 2002) is widely used to describe the strength of rocks and jointed rock masses. To use this failure criterion, a few parameters such as σ_c, s, m_i, a, and GSI (Geological Strength Index), are required. For intact rock strength, s = 1, a = 0.5, GSI = 100, only σ_c and m_i are needed. For jointed rock masses, σ_c, m_i, and GSI are needed. σ_c can be obtained from uniaxial compression tests. The m_i values are believed to vary with rock type, and it is recommended that these values be determined from a series of triaxial tests (Hoek & Brown 1980a). Quite often, triaxial tests are not routinely conducted for most projects and engineers are forced to determine the m_i values empirically.

Hoek (2007) provided some m_i values as shown in Table 1. Possible data ranges are shown by a variation range value immediately following the suggested m_i value. m_i values range from 4 to 33 for some commonly encountered rocks and an impression that m_i depends only on rock type can be seen from the table but this is not true. The m_i value depends on many factors such as mineral content, foliation, and grain size (texture). A trend which can be easily seen from the table is that the m_i values are high for coarse grained rocks, moderate for medium grained rocks, and low for fine grained rocks. For sandstones, the m_i values can vary between 13 and 21, and for slates, between 3 and 11, according to Table 1. Such a large variation range is expected for rocks but at the same time, it presents a major challenge for engineers to choose a reasonably accurate m_i value for a particular rock.

It can be shown that if the compressive to tensile strength ratio R is high (e.g. $R \geq 8$), m_i can be estimated from this strength ratio, i.e.,

$$m_i \approx \frac{\sigma_c}{|\sigma_t|} = R \tag{1}$$

Hence, if the tensile strength and uniaxial compressive strength are available, the strength ratio R could be used as a good estimate for m_i. In other words, because σ_c can be easily obtained and if σ_t can be accurately determined, the m_i parameter can be reasonably estimated.

In this study, we will examine the relationship between the uniaxial compressive strength and the tensile strength of rocks from a different perspective, by applying the Griffith's theory and using the characteristic stresses observed in the uniaxial compression test. A simple method to estimate σ_t and m_i using uniaxial compression test data is presented.

Table 1 Updated values of the constant m_i for intact rock, by rock group. Note that values in parenthesis are estimates (Hoek 2007)

Rock type	Class	Group	Texture			
			Coarse	Medium	Fine	Very fine
SEDIMENTARY	Clastic		Conglomerate* (22±3) Breccias (19±5)	Sandstone 17±4	Siltstone 7±2 Greywacke (18±3)	Claystone 4±2 Shales (6±2) Marls (7±2)
	Non-Clastic	Organic				Chalk 7±2
		Carbonates	Crystalline Limestone (12±3)	Sparitic Limestone (10±2)	Micritic Limestone(9±2)	Dolomites (9±3)
		Evaporites		Gypsum 8±2	Anhydrite 12±2	
METAMORPHIC	Non Foliated		Marble 9±3	Hornfels(19±4) Metasandstones 26±6	Quartzites 20±3	
	Slightly foliated		Migmatite (29±3)	Amphibolites 26±6		
	Foliated*		Gneiss 28±5	Schists 12±3	Phyllites (7±3)	Slates 7±4
IGNEOUS	Plutonic	Light	Granite 32±3	Diorite 25±5		
			Granodiorite (29±3)			
		Dark	Gabbro 27±3	Dolerite (16±5)		
			Norite 20±5			
	Hypabyssal		Porphyries (20±5)		Diabase (15±5)	Peridotite (25±5)
	Volcanic	Lava		Rhyolite (25±5) Andesite 25±5	Dacite (25±3) Basalt (25±5)	Obsidian (19±3)
		Pyroclastic	Agglomerate (19±3)	Breccia (19±5)	Tuff (13±5)	

* Conglomerates and breccias may present a wide range of m_i values depending on the nature of the cementing material and the degree of cementation, so they may range from values similar to sandstone to values used for fine grained sediments.

** These values are for intact rock specimens tested normal to bedding or foliation. The value of m_i will be significantly different if failure occurs along a weakness plane.

2 BACKGROUND

2.1 *Griffith's theory*

Griffith (1924) proposed that the failure of brittle materials is governed by the initial presence of microcracks. Under uniaxial tension, the tensile strength predicted by the Griffith's theory is

$$\sigma_t = \sqrt{\lambda E' \gamma / c} \tag{2}$$

where $E' = E$ for plane stress problems and $E' = E/(1-\nu^2)$ for plane strain problems and E is the Young's modulus, ν is Poisson's ratio, γ is the specific surface energy, c is the half crack length, and λ is a numerical constant ($\lambda = 2/\pi$).

Under uniaxial and biaxial compression, neglecting the influence of friction on the cracks when closed, and assuming that elliptical cracks will propagate from the points of maximum tensile stress concentration, a stress criterion is obtained as (Griffith 1924)

$$\begin{aligned} (\sigma_1 - \sigma_3)^2 - 8\sigma_t(\sigma_1 + \sigma_3) = 0 \quad &\text{if } \sigma_1 + 3\sigma_3 > 0 \\ \sigma_3 + \sigma_t = 0 \quad &\text{if } \sigma_1 + 3\sigma_3 < 0 \end{aligned} \tag{3}$$

where σ_1 and σ_3 are the two principal stresses. It should be noted that Griffith's original theory was intended for tensile strength and Eq. (3) was extended based on assumptions of a parabolic Mohr failure envelope. The primary assumption of this criterion is that macroscopic failure is identical to the initiation of cracking from the longest, most critically oriented crack. It can be seen from Eq. (3) that the Griffith stress criterion predicts a strength ratio of $R_G = \sigma_c/|\sigma_t| = 8$.

The original Griffith's theory was extended by Murrell (1963) into three dimensions to consider the triaxial state of stress near the tip of flat ellipsoidal cracks. The criterion is written as

$$(\sigma_1 - \sigma_3)^2 + (\sigma_2 - \sigma_3)^2 + (\sigma_3 - \sigma_1)^2 - 24\sigma_t(\sigma_1 + \sigma_2 + \sigma_3) = 0 \tag{4}$$

Murrell's failure criterion predicts that the rock compressive strength equals 12 times the tensile strength (i.e. Murrell's strength ratio is $R_M = 12$).

The Griffith strength ratios are smaller than generally observed for rocks but is of the correct order. The reason why the Griffith criteria fail to predict the observed strength ratio of rocks is because the theories are only dealing with the initiation of failure (crack initiation) whereas the strength observation refer to the final or macroscopic failure (Paterson & Wong 2005). As will be discussed in the next section, under tensile conditions, the crack initiation is often equivalent to total failure. Under compressive conditions, crack initiation happens at a stress level normally much lower than the peak stress. Crack growth under compressive loading is stable and additional load is required to increase the crack length.

2.2 *Microfracturing, damage, and failure of brittle rocks*

Direct tension tests can be conducted to observe the crack initiation and propagation process in tension and to determine the tensile strength of rocks. Because tensile fracturing under tensile loading is unstable, observation of the fracturing process is difficult. Despite the difficulty, numerous direct tension tests have been conducted and meaningful results obtained. It is seen from the test results that the tensile failure process of rock, which includes initiation, growth, and propagation of micro-fractures, happens when the axial stress is close to the peak strength. For example, the crack initiation and propagation stresses of norite under uniaxial tension are 95% and 97% of the peak tensile strength, respectively (Bieniawski 1967). Hence, under overall tensile loading, the stress levels of crack initiation ($(\sigma_{ci})_t$) and crack damage ($(\sigma_{cd})_t$, which here is defined as the stress level at which crack coalescence occurs) are very close to the peak strength (σ_t) so that we can write

$$(\sigma_{ci})_t \approx (\sigma_{cd})_t \approx \sigma_t \tag{5}$$

Several characteristic stress levels have been identified from laboratory uniaxial compression tests. In the stress-strain relation shown in Figure 1, σ_{cc} is the crack closure stress level, σ_{ci} is the crack initiation stress level, and σ_{cd} is the crack damage stress level which is close to the long-term rock strength (Bieniawski 1967, Cai et al. 2004, Martin 1993, Martin 1997). AE (acoustic emission) rates start at σ_{ci} and increase drastically when σ_{cd} is reached. The maximum AE rate is usually reached near the peak strength σ_c. The three stress levels, i.e., σ_{ci}, σ_{cd}, and σ_c, represent important stages in the development of the macroscopic failure process of intact rocks.

Microscopic observations indicate that newly generated cracks are tensile in nature, generated by extension strain, and mostly aligned in the same direction as the maximum compressive stress. After the crack initiation, the propagation of the microcracks (often called as Griffith cracks) is a stable process, which means that the cracks only extend by limited amounts in response to given increments in stress.

Crack initiation starts at stress levels of approximately 1/3 to 2/3 times the peak uniaxial compressive strength for most brittle rocks (Brace et al. 1966, Bieniawski 1967, Martin 1997, Cai et al. 2004). In laboratory tests on intact rocks, the crack initiation stress or threshold is defined by the onset of stable crack growth or dilatancy, which can be identified from the stress – volumetric strain curve as the point of the departure of the volumetric strain observed at a given mean stress from that observed in hydrostatic loading to the corresponding pressure (Bieniawski 1967).

As can be seen in Figure 1, both the volumetric strain and the crack volumetric strain plots can be used to identify the crack initiation stress level σ_{ci}. σ_{ci} can also be defined as the point where the volumetric strain starts to deviate from the straight line in the elastic deformation stage (Stage II) or the crack volumetric strain deviates from zero, as shown in Figure 1. AE starts to appear systematically when the stress level is above σ_{ci}. AE monitoring is an alternative tool to determine the crack initiation stress but a clear boundary between background noise and true crack initiation is difficult to be identified sometimes. The volumetric strain or the crack volumetric strain plot provides a more objective approach for determining the crack initiation stress, especially under uniaxial compressive loading condition.

σ_{ci}/σ_c is an indicator of rock heterogeneity and texture. A low σ_{ci}/σ_c value indicates high rock heterogeneity (Martin 1993, Cai et al. 2004). For heterogeneous rocks such as coarse granite, the crack initiation stress level could be as low as 0.3. For fine grain dolomite, the crack initiation stress level could be as high as 0.6 (Cai et al. 2004). In the following discussion, σ_{ci} is taken, along with σ_c and Griffith's theory, to link the tensile strength to the uniaxial compressive strength of rocks.

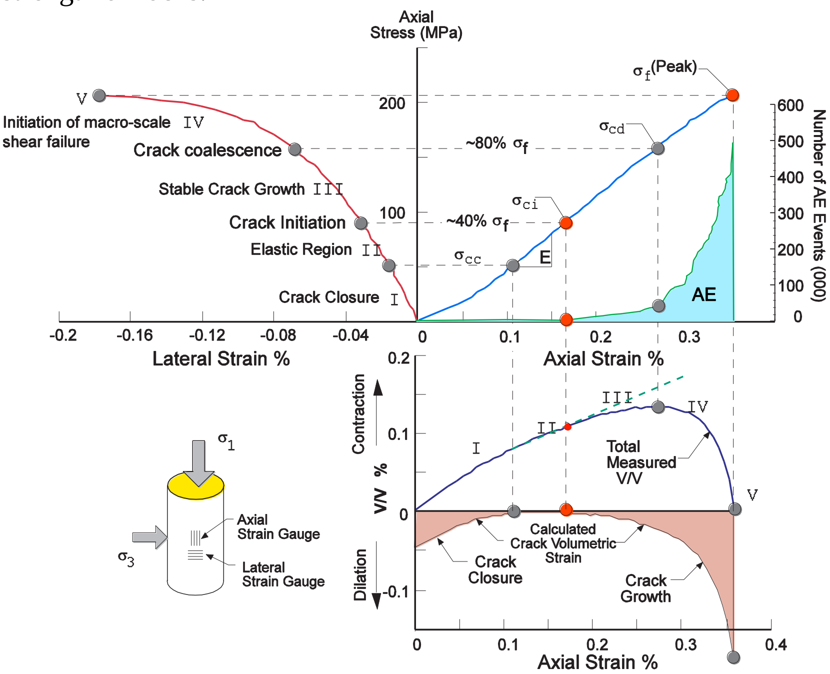

Figure 1. Stress-strain diagram of a granite showing the stages of crack development (after (Martin 1993)).

3 ESTIMATE OF TENSILE STRENGTH AND HOEK-BROWN PARAMETER m_i OF ROCKS FROM UNIAXIAL COMPRESSION TESTS

3.1 *Tensile strength of strong brittle rocks*

In the light of the observations discussed in the previous sections, it is evident that when compared to its tensile strength, a rock tends to have a much higher compressive strength because microcrack initiation (tensile in nature), growth and propagation in compression is stable and additional load is required to cause further crack growth. In contrast to stable crack propagation in compression, microcrack growth and propagation in a pure tensile stress field is unstable. In tension, the crack initiation stress is very close to the peak tensile strength. In compression, the crack initiation stress is lower than the peak uniaxial compressive strength so that a modification of the Griffith stress criterion assumption is necessary in order to predict the correct strength ratio ($\sigma_c/|\sigma_t|$).

Crack initiation in tensile loading often means tensile fracture is imminent. On the other hand, crack initiation in compression is only linked to a lower stress as compared to the peak compressive strength. When both crack initiation and peak strength are considered the same (e.g. Griffith's approach), it leads to a $\sigma_c/|\sigma_t|$ ratio of 8. However, additional loading is required to bring the stress level from σ_{ci} to σ_c in compression. The ratio of σ_c/σ_{ci} indirectly reflects the gap between crack initiation and peak strength in compression. Hence, it is proposed to estimate the tensile strength of strong brittle rocks from the crack initiation stress (σ_{ci}) and the uniaxial compressive strength (σ_c) using the strength ratio from the following equation

$$R = R_G \frac{\sigma_c}{\sigma_{ci}} = 8\frac{\sigma_c}{\sigma_{ci}} \tag{6}$$

Here, the constant R_G = 8 is adopted from the Griffith stress criterion (see Eq. (3)). As stated before, the Griffith stress criterion applies to crack initiation only so that $\sigma_{ci}/\sigma_t = 8$ (i.e., $\sigma_c = \sigma_{ci}$ holds true when applying Griffith stress criterion). In reality, σ_{ci} and σ_c are different. In our proposed approach, σ_c/σ_{ci} is used to reflect the difference between the crack initiation stress and the peak strength in compression.

For coarse grained rocks such as coarse granite and diorite, σ_{ci}/σ_c is usually between 0.3 and 0.4. Hence, the strength ratio R would usually be between 20 and 27 according to Eq. (6). For medium grained dolerite and sandstone, σ_{ci}/σ_c is usually between 0.4 and 0.5 and the strength ratio R will be in the range of 16 to 20. For fine grained siltstone, limestone, and sandstone, σ_{ci}/σ_c is usually between 0.5 and 0.7 so that R is between 11 and 16. These strength ratios are close to those obtained from laboratory tests.

It follows that the tensile strength of the rock can be obtained from

$$|\sigma_t| = \frac{\sigma_c}{R} = \frac{\sigma_{ci}}{8} \tag{7}$$

3.2 *Hoek-Brown parameter m_i of strong brittle rocks*

For strong brittle rocks, the Hoek-Brown parameter m_i can be approximated by R, i.e.,

$$m_i \approx R = 8\frac{\sigma_c}{\sigma_{ci}} \tag{8}$$

The range of values of σ_{ci}/σ_c for coarse, medium, and fine grained brittle rocks are 0.3 – 0.4, 0.4 – 0.5, 0.5 – 0.6; accordingly, m_i values predicted by the proposed method are 20 – 27, 16 – 20, and 11 – 16, respectively, generally in good agreement with most data listed in Table 1. The overall good agreement of the results from our method with those obtained from experiments and suggested by Hoek is by no means accidental. Our proposed method, based on the assumptions regarding the failure envelopes in tension and compression according to Griffith's work, reveals an important linkage among σ_{ci}, σ_c, σ_t.

However, detailed analyses of some published data revealed that the estimated m_i (by using $m_i \approx R$) can deviate from the tested or the suggested Hoek-Brown parameter m_i in Table 1.

This can be attributed to how this parameter is obtained. Strictly speaking, the Hoek-Brown parameter m_i has to be obtained from a series of triaxial compression tests. Although it has been shown that m_i is roughly the strength ratio R (see Eq. (1)), existing test data were mostly in the compression zone. The uniaxial compressive and tensile strengths from the fitted Hoek-Brown envelope may not be exactly the same as the tested ones. In general, the error in uniaxial compressive strength estimation by triaxial data fitting is small as compared to that for the tensile strength. Test data by Carter et al. (1991) show that for potash rock and limestone (soft rocks), the fitted Hoek-Brown curves overestimate the tensile strength of the rocks, but for granite (strong and brittle rocks), the curve fitting underestimates the tensile strength (Table 2). Test data by Alber and Heiland (2001) on limestone show that the tensile strength inferred from the fitted curve in the compression zone consistently overestimates the true tensile strength. The statement expressed by Eq. (1) is true only if the fitted σ_c and σ_t are used (columns 5 and 6 in Table 2), not the test values (columns 2 and 3 in Table 2).

Table 2 Test data and fitting result summary (data from (Carter et al. 1991))

(1)	(2) Tested σ_c (MPa)	(3) Tested σ_t (MPa)	(4) R (test data)	(5) H-B σ_c (MPa)	(6) H-B σ_t (MPa)	(7) H-B m_i
Potash rock	25	2	12.5	25.5	6	4
Limestone	52	4	13	57	6.5	8.6
Lac du Bonnet granite	226	13	17.4	249	8.5	29

As was concluded by Carter et al. (1991), fitting triaxial test data using the Hoek-Brown failure criterion is very good for hard, strong crystalline rocks. The fit is especially good at high confining pressure, but much less so in the low confining pressure and tension zone. Hence, we consider that the m_i value is not a constant but confining pressure dependent. As shown in Figure 2, two different m_i values, $(m_i)_t$ for the tension zone and $(m_i)_c$ for the compression zone, can be used in the Hoek-Brown failure criterion.

When a rock is subjected to a high confining stress, microcrack initiation from most pre-existing defects in the rock will follow the stress state defined by 3D Griffith ellipsoidal cracks (Murrell 1963). It is observed that the estimated m_i from Eq. (8) provides a m_i value for the tension to low confinement zones (e.g. $\sigma_3 < 5$ MPa, for rocks near the underground excavation boundary, which are of a concern for excavation stability), since all cracks are assumed to follow 2D Griffith's stress state. Under a high confining stress state, ellipsoidal microcrack initiation and propagation will dominate and hence Murrell's strength ratio R_M, instead of Griffith's strength ratio R_G, is considered more appropriate for the estimation of m_i. It is proposed to estimate the m_i value for the high confinement zone from

$$m_i = (m_i)_c \approx R_M \frac{\sigma_c}{\sigma_{ci}} = 12 \frac{\sigma_c}{\sigma_{ci}} \tag{9}$$

where the subscript "c" in $(m_i)_c$ indicates the m_i value for the high compression zone. Hence, the m_i value for the high compression zone is 1.5 times larger than the value (Eq.(8)) for the tension zone (and low confinement zone) for strong brittle rocks. It can be seen from a few examples below and in the next section that the m_i values obtained from Eq. (9) are close to the triaxial test values of strong crystalline rocks.

Test data in the tension and low confinement zone in Figure 2 are from Johnson et al. (1987) for Westerly granite. The Hoek-Brown failure envelope in the compression zone in Figure 2 was based on σ_c = 214 MPa, m_i = 26.7, obtained from test data by Heard et al. (1974) (cited in (Hoek & Brown 1980b)). The Hoek-Brown failure envelope that fits well to the test data by Johnson et al. (1987) is defined by σ_c = 190 MPa, m_i = 14. Clearly, this m_i value is smaller than the one obtained from the triaxial data in the compression zone. If the Hoek-Brown envelope, defined by σ_c = 214 MPa, m_i = 26.7, is extended to the tension zone (curve A-B in Figure 2), we find that the tensile strength of the rock is underestimated ($|\sigma_t|$ = 8 MPa). If we fix σ_c = 214 MPa and change m_i to fit the test data by Johnson et al. (1987), then, m_i = 16 results (as indicated by the blue line (curve B-D) in Figure 2), which is very close to 17.8, a value obtained by dividing $(m_i)_c$ (= 26.7) by 1.5.

In summary, the m_i values estimated using $m_i \approx 8\sigma_c / \sigma_{ci}$, based on 2D Griffith's crack model, need to be considered as the lower bound, applicable to tension and low confinement zones for strong brittle rocks. Since m_i is traditionally determined and applied for triaxial stress states in compression to a confinement range of 0.5 to 1.0 of σ_c, the upper bound estimation using $m_i \approx 12\sigma_c / \sigma_{ci}$ provides a better match to the triaxial test data.

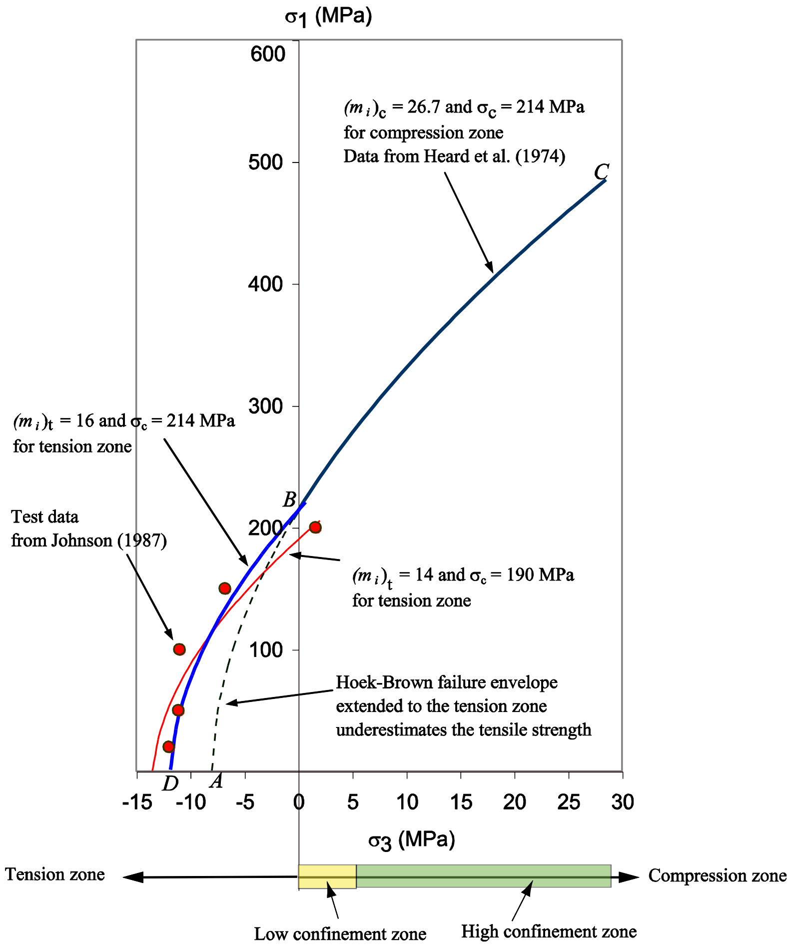

Figure 2. m_i values for tension and compression zones for the Westerly granite. $(m_i)_t$ and $(m_i)_c$ are m_i values for tension ($\sigma_3 < 0$) and compression ($\sigma_3 > 0$) zones, respectively.

3.3 *Application examples*

3.3.1 *Medium to coarse grained granite from the Mine-by tunnel*

The Lac du Bonnet granite from the Mine-by tunnel in Canada is among the most thoroughly investigated rocks in the rock mechanics community, both in the laboratory and in situ. The mechanical properties of the medium to coarse grained pink granite are: elastic modulus – 65±5 GPa, Poisson's ratio – 0.25±0.05, uniaxial compressive strength – 213±20 MPa, tensile strength (obtained from Brazilian test) – 8.9±1 MPa, crack initiation stress 70 to 80 MPa (Martin 1993, Read & Martin 1996, Diederichs 1999). According to Eq. (6), the strength ratio R is found to vary between 21 and 24. For comparison, the average strength ratio from the test data is R = 213 / 8.9 ≈ 24. If only the uniaxial compression test is conducted, we can estimate the tensile strength as σ_{ci} / 8 = (70 to 80) / 8 = 8.75 to 10 MPa with an average of 9.4 MPa, the Hoek-Brown strength parameter m_i as 34 and 23 for the compression and tension zones, respectively. The m_i value for the compression zone is very close to the triaxial compression test data listed in Martin and Stimpson (1994). The m_i values from the test data for the medium to coarse grained granite are 30.8 from the shallow ground and 34.8 from the 240 m level.

3.3.2 *Medium grained granite of Lac du Bonnet*

The microscopic fracture process in the Lac du Bonnet granite was investigated by Lajtai (1998) using cyclic loading. The test samples were obtained from the Cold Spring Quarry near Lac du Bonnet, Manitoba, Canada.

The mechanical properties of the granite are: elastic modulus – 70 GPa, Poisson's ratio – 0.21, uniaxial compressive strength – 225 MPa, tensile strength (from Brazilian test) – 13.5 MPa. The crack initiation stress of the granite studied by Lajtai is 100 MPa (Lajtai 1998). According to Eq. (6), the strength ratio is $R = 8 \times 225 / 100 = 18$, which is very close to the test result of $225 / 13.5 \approx 17$. Accordingly, the tensile strength σ_t and the Hoek-Brown strength parameters m_i for this rock can be estimated as 12.5 MPa, and 27 (compression zone), and 18 (tension zone), respectively from the uniaxial compression test. 412 triaxial tests of the granite at 13 confining pressures (the maximum confining pressure was 40 MPa) were conducted by Carter et al. (1991). The Hoek-Brown parameter m_i from the test data was 29. The estimate of $(m_i)_c = 27$ is very close to the test data.

3.3.3 *Test data from a mine site*

Laboratory tests were carried out at the Geomechanics Research Center of Laurentian University in Canada, using rock samples from a mine site in Canada and the test results are summarized in Table 3. In each uniaxial compressive strength test, two strain gauges were installed to measure the axial and lateral strains. Brazilian tests were used to obtain the tensile strength of the rocks. Two Brazilian test samples were cut right adjacent to the sample used for the uniaxial compressive strength test to ensure the comparability of the obtained σ_t to σ_c. Loading directions of the two Brazilian samples were perpendicular to each other. The tensile strength that is corresponding to a particular UCS sample is obtained by taking the average of the two Brazilian test results. In general, the tensile strengths in the two orthogonal directions are very close but in one case (Sample-11), there is a large difference in the tensile strengths of the two Brazilian tests. It is believed that this is due to the influence of weak planes in the sample.

In the uniaxial compression tests, the crack initiation stresses are identified from the volumetric strain plots. Figure 3 shows the stress-strain relationships for Sample-2. The peak UCS strength is 83 MPa. No crack closure stress can be identified from the plot. The crack initiation stress is identified from the volumetric strain plot as 42 MPa. The estimated m_i parameters and tensile strength are 16 (tension zone), 24 (compression zone), and 5.25 MPa, respectively. The average tensile strength from the Brazilian test is (6.6+4.8)/2 = 5.7 MPa. In this case, the error in tensile strength estimation by the proposed method is 7.9%.

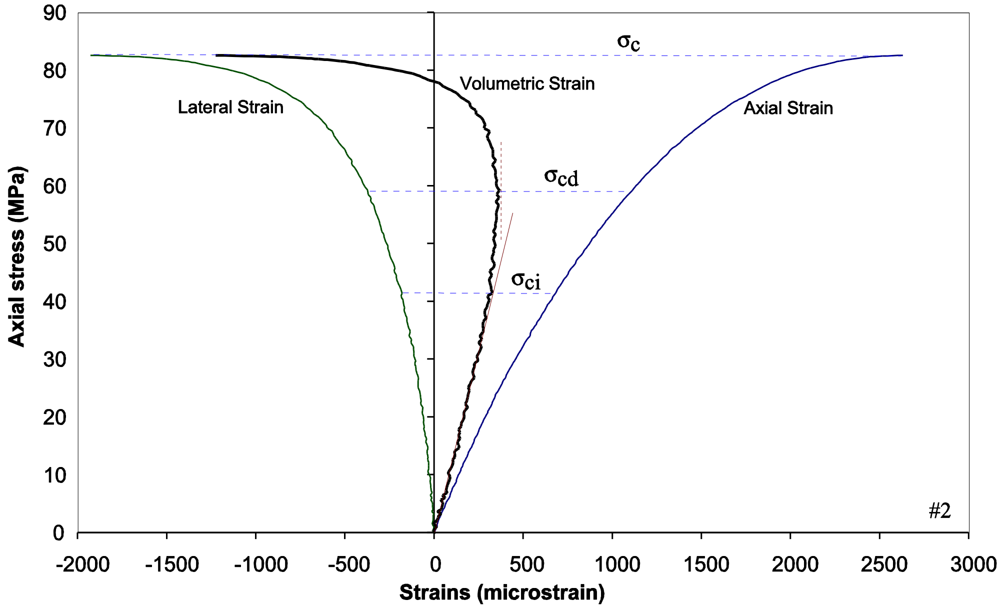

Figure 3. Stress-stain relationship of rock Sample - 2.

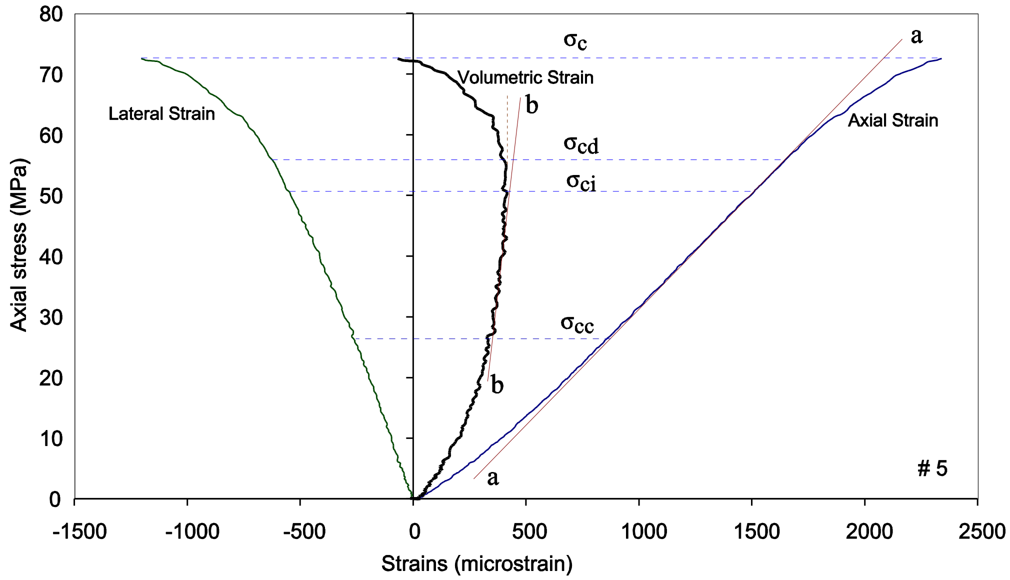

Figure 4. Stress-stain relationship of rock Sample - 5.

Table 3 Test result summary and tensile strength and m_i value prediction for a group of rocks from mine site in Canada

#	Rock type[1]	σ_c (MPa)	σ_{ci} (MPa)[2]	σ_{ci}/σ_c	*Strength ratio R*[5]	$(m_i)_c$ [6]	Predicted σ_t (MPa)	Brazilian σ_t (a) (MPa)[3]	Brazilian σ_t (b) (MPa)[4]	Average σ_t (MPa)	Error in σ_t prediction (%)
1	sumx	70	33	0.47	17.0	25.5	4.13	5.7	3.2	4.45	7.3
2	sumx	83	42	0.51	15.8	23.7	5.25	6.6	4.8	5.7	7.9
3	sumx	91	53	0.58	13.7	20.6	6.63	6.2	7.4	6.8	2.6
4	sumx	115	43.5	0.38	21.1	31.7	5.44	4.9	5.4	5.15	5.6
5	sch	65	52	0.80	10.0	15.0	6.50	8.6	8.9	8.75	25.7
6	sch	69	54	0.78	10.2	15.3	6.75	9.3	10.5	9.9	31.8
7	sch	73	51	0.70	11.5	17.2	6.38	9.4	8	8.7	26.7
8	qte	152	78	0.51	15.6	23.4	9.75	10.1	12.7	11.4	14.5
9	prdt	67	47	0.70	11.4	17.1	5.88	7.7	4.7	6.2	5.2
10	prdt	155	75	0.48	16.5	24.8	9.38	9.6	7.1	8.35	12.3
11	peg	169	82	0.49	16.5	24.7	10.25	8.6	15.1	11.85	13.5
12	masu	124	84	0.68	11.8	17.7	10.50	13	10.7	11.85	11.4
13	ampt	110	64	0.58	13.8	20.6	8.00	10.5	7.3	8.9	10.1

Note: 1. sumx – sulphide mix; sch – schist; qte – quartzite; prdt – peridotite; peg – pegmatite; masu – massive sulphide; ampt – amphibolites. 2. Determined from volumetric strain plots. 3. The Brazilian samples are cut right adjacent to the UCS sample. 4. The loading direction in Sample – b is perpendicular to the loading direction in Sample – a. 5. $(m_i)_t$ for tension zone $(8*\sigma_c/\sigma_{ci})$. 6. $(m_i)_c$ for compression zone $(12*\sigma_c/\sigma_{ci})$.

For some rock types, the initial stage for crack closure is characterized by a prolonged concave portion in the axial stress – axial strain plot (Figure 4). We need to first identify the linear elastic response from the axial stress – axial strain plot by drawing line a-a. This will define the crack closure stress. Line b-b is then drawn starting from the crack closure stress on the volumetric strain plot. The deviation point that marks the crack initiation stress is 52 MPa. The peak UCS strength of the sample is 65 MPa. The estimated m_i parameter and tensile strength are 10 (tension zone), 15 (compression zone), and 6.5 MPa, respectively. The average tensile strength from the Brazilian test is (8.6+8.9)/2 = 8.75 MPa. The error in tensile strength estimation by the proposed method is 25.7%, which is slightly high but acceptable for tensile strength estimate of schist. For comparison, the m_i values suggested in Table 1 is 12±4 for schist.

The estimated tensile strength and m_i values are listed in Table 3 for other tested samples. For most cases, the error in tensile strength estimate is within 15%, in line with the uncertainty range found by Brace et al. (1966) for determining σ_{ci}. The error is only found to be large for schist which has higher crack closure and crack initiation stresses (σ_{ci}/σ_c = 0.7 to 0.8). The existence of weak seams in schist is also responsible for the large error in tensile strength prediction.

3.4 *Discussion*

It is noted that the strength ratios R for both the pink granites and gray granites (granodiorites) from the URL are very close to the test data but the m_i values estimated by $(m_i)_t \approx R$ are smaller than the values suggested in Table 1. In fact, they are even smaller than the lower limits suggested for granites (29) and granodiorites (26). m_i values in Table 1 are obtained from triaxial tests which usually apply very high confinement stresses and are hence are applicable to the high confinement zone. The strength ratio R cannot be used to estimate accurately the m_i value for the strength envelope in the high confinement zone. Estimation of $m_i \approx R$ can only be used in the tension zone (and low confining pressure zone). On the contrary, the m_i values estimated by $(m_i)_c \approx 12\sigma_c/\sigma_{ci}$ are very close to the test data and the values suggested in Table 1. It indirectly proves that the logic of using 3D Griffith crack model under high confinement condition is correct. Hence, $m_i \approx 12\sigma_c/\sigma_{ci}$ is a good estimate for the Hoek-Brown parameter m_i for strong crystalline rocks under high confining stress conditions. Data from Bell and Jermy (2000) also support this notion. The average uniaxial compressive strength of the dolerites is 169.2 MPa (120 samples) and the average tensile strength is 12.4 MPa. The strength ratio is 13.6. The m_i value for the tension zone is 13.6 according to $(m_i)_t \approx R$. However, the average m_i value from 35 triaxial tests is 18.3, higher than what is obtained from $(m_i)_t \approx R$. If we apply the methodology described here, the average m_i value for high confinement zone would be $1.5 \times 13.6 \approx 20$, a better approximation of the test value.

Figure 5 present the Hoek-Brown strength envelopes of strong brittle rocks with m_i = 20 and 30. The lower value of 20 defines the tensile zone and the low confinement zone. The higher value of 30, which is 1.5 times of the lower value of 20, is applicable to the high confinement condition. We can see that there must be a transition zone in the figure that changes the m_i = 20 strength envelope to the m_i = 30 strength envelope gradually. This transition zone is illustrated in the figure in an area confined by lines σ_1/σ_3 = 20 and 30, and Hoek-Brown strength envelopes defined by m_i = 20 and 30. It is seen that for brittle strong rocks with σ_c = 100 to 200 MPa, the confinement which divides the “low” and “high” confinement zones is between 5 to 10 MPa. This observation is in agreement with Paterson and Wong (2005), who state that that below 10 MPa confining pressure, the stress maximum is still associated with proliferation and growth of predominantly axial microcracks but the macroscopic longitudinal slabbing that tends to develop in uniaxial tests is suppressed.

σ_1/σ_3 = constant is called the spalling limit by Kaiser et al. (2000). Spalling limit of rocks defines the boundary over which the stress ratio σ_1/σ_3 is greater than, the rock will fail predominantly in the mode of spalling and slabbing caused mainly by the tensile fracture propagation under low confinement condition. The spalling limit depends on rock heterogeneity, and for less heterogeneous intact rocks it can be higher than 30. In general, σ_1/σ_3 = 20 to 30 is applicable to some brittle rocks. When we consider the mechanism of crack propagation under different confinement condition, we observe that the spalling limit for intact rocks can be explained as the transition from 2D Griffith crack propagation under low confinement condition to 3D Griffith crack propagation under high confinement condition. When the transition zone is considered, the strength envelope becomes multi-segmented. This type of multi-segmented strength envelope had been suggested by Kaiser et al. (2000) but our explanation of 2D to 3D Griffith’s crack deformation behavior transition in combination with low and high m_i values for low and high confinement zones provides a new dimension in understanding this type of failure criterion.

It should be noted that the strength envelope presented in Figure 5 may present difficulty in numerical modeling because the discontinuity on the gradient and the concave in the zone of transition, respectively. These problems should be addressed when one tries to employ these strength envelopes in numerical modeling. In addition, the multi-segmented (Figure 5) strength envelopes are only applicable to some particular rocks and cannot be generalized.

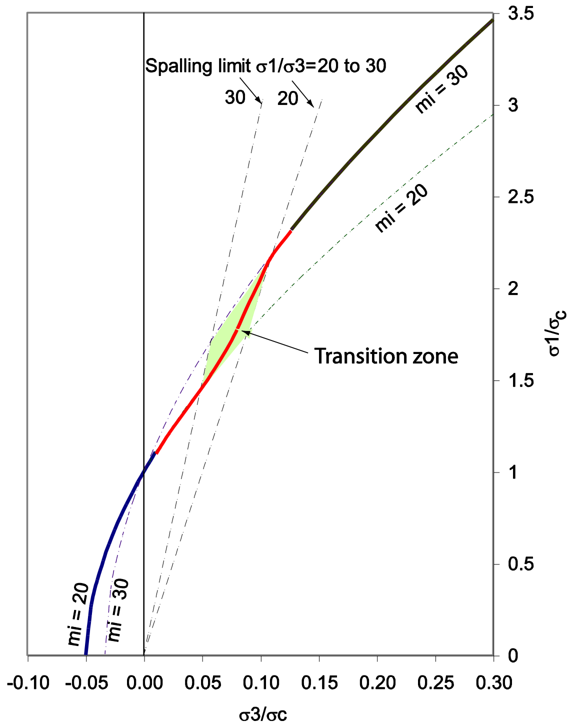

Figure 5. m_i value change and the spalling limit.

4 CONCLUSIONS

The tensile strength σ_t of strong brittle rocks can be linked to the crack initiation stress σ_{ci} and the peak strength σ_c in uniaxial compression tests using the strength ratio of $R = \sigma_c / |\sigma_t| = 8\sigma_c / \sigma_{ci}$. The original Griffith's theory predicts well the crack initiation in both tension and compression. However, crack growth in tension is unstable while stable in compression. The ratio of σ_c / σ_{ci} is employed to account for the difference of crack growth or propagation in uniaxial tension and compression.

When the crack initiation stress is determined by AE monitoring or volumetric strain measurement, the tensile strength can be estimated by $|\sigma_t| = \sigma_{ci} / 8$. The proposed method, which reveals an important linkage among σ_t, σ_c, and σ_{ci}, is validated using laboratory test data. The predicted tensile strengths are in good agreement with the test values, with error generally less than 15%.

The Hoek-Brown strength parameter m_i depends on confining pressure. For practical estimate of m_i, it is found that $m_i \approx 12\sigma_c / \sigma_{ci}$ can be applied to strong, brittle rocks (e.g. crystalline rocks), applicable to high confinement zone. For low confinement to tension zone, especially for the tension zone, $m_i \approx 8\sigma_c / \sigma_{ci}$ can be used.

Rock type cannot be used directly to define the strength ratio R and the m_i value. Data inferred from the databases can only be used when there are no test data available at the initial design stage. Whenever possible, laboratory tests should be conducted to determine the tensile strength and the m_i values more accurately. The method suggested in this paper provides an easy and yet accurate way to determine these two important parameters for brittle rocks from conventional uniaxial compression tests.

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