Fracture Mechanics Numerical Modeling – Potential and Examples of Applications in Rock Engineering

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ABSTRACT: In fracture mechanics modeling the rock mass is seen as the combination of rock material and fractures, which are evident on different scales. The activation and propagation of fractures dominates the stability and performance issues as well as fluid flow potential. Once fractures have started to grow into each other and create through going separations, stability of rock engineering constructions may be violated. Further, as fracture mechanics inherently incorporated a scaling law in the form of the participating fracture length, scaling issues can be more accurately mirrored. This contribution highlights some examples of application of a fracture mechanics based code on some rock mechanics issues such as numerical laboratory experiments, borehole stability or the creation of fluid pathways due to underground constructions. The results show explicitly the creation of fractures and connection of existing ones, and hence the safety and performance issues. It has to be concluded that the basis for frequent application of fracture mechanics on specialized challenges in Geomechanics is laid, and the needs for future developments are defined.

1 INTRODUCTION

In classical rock mechanics rock is viewed as a continuous flawless material. Failure or strength of rock is described by rather empirical criteria, which depend on the phenomenological description of mostly laboratory data. There exist numerous criteria out of which probably the Mohr-Coulomb, the Hoek-Brown, the Mogi, and the Lade Duncan criterion and modifications or extensions of those are the most frequently used (Benz & Schwab, 2008).

The parameters to the strength criteria are determined in laboratory geotechnical testing, where the response is characterized only globally, i.e., based on measurements at the boundary that are assumed representative of the overall (homogeneous, continuous) sample response. Thus, only in case of a perfectly homogeneous material undergoing perfectly uniform deformation, a constitutive behavior may be expected. However, the hypothesis of continuity does not hold for rock. Rock material is a discontinuous combination of solid matter, pores, cracks and fractures. The non-homogeneous characteristics of rock in its geomechanical behavior has been studied at different scales and by different techniques, like acoustic emission analysis (e.g. Lockner 1995) or 3D X-ray computed tomography (CT) (e.g. Desrues et al. 2006).

All analyses of the structural breakdown of rock clearly show that a continuous approach cannot reflect the mechanics of breakdown correctly. The existence of a crack in an otherwise solid homogeneous body reduces the strength of the structure considerably. Any load acting on the body is magnified several times at the tip of such a discontinuity and when the stress concentration at the tip of the crack reaches a critical level, it propagates. When rock is loaded, a swarm of pre- existing discontinuities redistributes the stresses locally and individual cracks may start propagating. When stressed further, isolated reactivated microcracks may coalesce and form a larger fracture.

Linear fracture mechanics provides the tools to estimate the stress and displacement fields around the tip of a crack. Any loading of a fracture will result in an alteration of the stresses at the fracture tip, which may be described by the stress intensity factor K_k, which is a function of the crack length describing the shape and grade of stress concentration a the crack tip (Irwin, 1960). The resistance of rock to the propagation of fractures is described in terms of fracture toughness. The fracture toughness K_{kC} is the limit of local stress increase, i.e. stress intensity, due to an existing fracture at onset of critical extension. A comprehensive introduction to fracture mechanics and related laboratory determination of key parameters and may be found elsewhere (e.g. Whittaker et al., 1992; Backers, 2005).

Therefore, discontinuities, i.e. cracks, pores, fractures or joints, are an important feature of rock and rock mass and control not only the hydraulic properties of rock by well-connected fracture networks (Quesada et al., in press), they also govern the mechanical behavior. When the stresses in a rock mass are altered, be it e.g. by the depletion of a reservoir, sequestration of liquid or gas, increase of pressure by EOR operations, or the introduction of a new excavation, the pre-existing discontinuities may grow. This may create new pathways for fluid flow or, if pre-existing fractures are propagated and coalesce to form larger structures that intersect with an excavation or another free surface, the structures may loose integrity and fail.

Due to the complex nature of rock mass behavior and related fracturing, a numerical modeling approach to the task is favorable. In this contribution the capability of using fracture mechanics based numerical simulations to rock mechanics applications is tested on a variety of examples. A selection of rock fracture mechanics modeling codes is outlined and the perspectives and future potentials are discussed.

2 NUMERICAL METHODS FOR FRACTURE SIMULATION

Several numerical approaches have been applied to rock mechanics challenges in general. For a review of the different methods and a comprehensive reference base refer to Jing and Hudson (2002) and in particular Jing (2003). For explicit fracture propagation analysis of geomaterials only few codes are available; see Bäckström et al. (2008) also. Here only selected approaches are discussed in general; completeness of arguments and methods is not assumed.

2.1 *Particle Flow Code*

A specific assumption about the basic composition of the material at an appropriate fundamental level of detail is made in the particle flow code (PFC). The rock material is represented as an assembly of bonded particles. The complex evolution of emergent failure patterns is then reduced to an explicit integration of the motion of each particle, subjected to the evolving forces imposed by neighboring particles. Fracture mechanics models and laboratory experimental input data can be employed limited only, and excessive tuning of the model is necessary, which limits the applicability of the method (Yoon, 2008).

2.2 *Boundary Element Method*

The boundary element method (BEM) has been applied frequently to rock fracturing and the basis of those codes are fracture mechanics principles. The fundamental building block is the discontinuity rather than a particle (c.f. PFC) and fracturing is modeled as a chain of cracks. The path of crack growth is almost independent of discretization in BEM, this in contrast to conventional finite element (see below). BEM is well suitable for large domains, however, it is limited in complexity of the model in terms of non-linear material behavior, inhomogeneity, and anisotropy. Examples of such approaches are Fracod2d (Shen et al., 2004) and DIGS (Napier and Backers, 2006).

Fracod2D is a two-dimensional BEM code that makes use of the Displacement Discontinuity Method (DDM). Fracture propagation is defined by a modified Energy Release Rate criterion (F- criterion, Shen and Stephansson, 1993), and fracture initiation is defined by a stress/strength comparison using the Mohr-Coulomb approach. While Fracod2D assumes fracture initiation at any location by a stress/strength comparison, DIGS considers the problem region to be covered by a random mesh of potential crack segments.

2.3 *Finite and Extended Finite Element Method*

Geomechanical tasks are generally of high geometrical complexity. A vast amount of publications on numerical modeling using finite element models (FEM) to rock engineering applications is available. The FEM is widely applied because of its flexibility in handling material heterogeneity, anisotropy, boundary conditions and non-linear material behavior. Many commercial codes are readily available and almost any physical law can be implemented in such codes with limited amount of effort (see e.g. Comsol Multiphysics).

However, if applied to the simulation of fracture propagation, the FE approach bares the limitation that crack propagation requires a doubling of the edges and subsequent adaptive re-meshing of underlying finite element mesh. This lead to a considerably increased computing time and numerical implementation effort, and the fracture propagation path is meshdependent.

The extended finite element method (XFEM) is an approach, which has the capacity to remove the mentioned limitation for the FEM. The XFEM has been developed in recent years only (e.g. Sukumar and Prévost, 2003; Budyn et al., 2004). The XFEM does not only remove the necessity of re-meshing, it locally improves the accuracy of the numerical solution considerably over that of the FEM, by introducing a priori knowledge about the near crack tip stress and displacement fields into the method (Schroeder, 2008).

Byfut et al. (2009) have introduced a new XFEM code, where the principles of h- and p- approaches are applied. The finite element method converges exponentially fast when the mesh is refined using a suitable combination of h-refinements (dividing elements into smaller ones) and p-refinements (increasing their polynomial degree) (e.g. Babuska and Guo, 1992; Schroeder, 2008). For a mathematical discussion of the matter refer to the mentioned publications and references given therein. Figure 1 shows how the combination of h- and p- methods not only improves the accuracy of such numerical solutions, but also reduces the computing time orders of magnitude. These developments give way in future to accurate near field analysis of fracture coalescence.

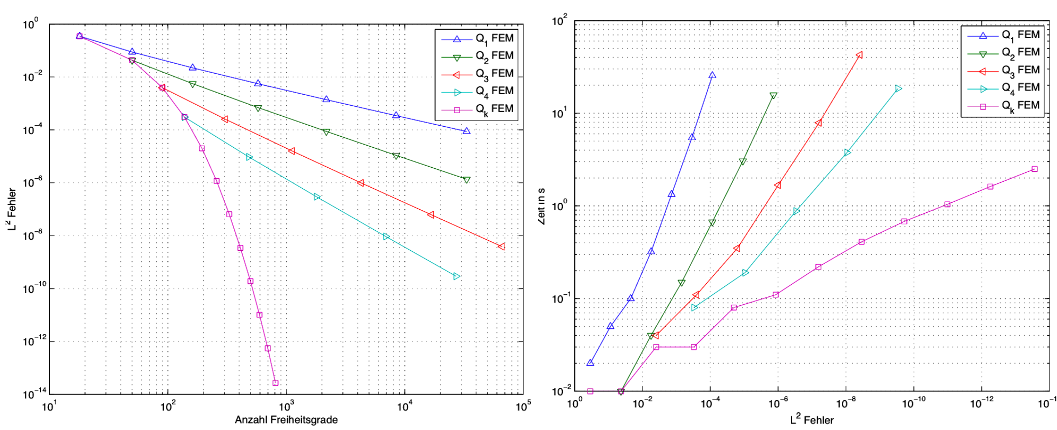

Figure 1. Using a hp- approach in XFEM improves the accuracy considerably at simultaneous high computing speeds. Comparing the numerical to the analytical solution, (left) the L2- error is less for a successively increased polynomial degree at constant mesh density (Qk; pink) compared to a constant polynomial degree with increased mesh density (Q1;:::;Q4), and (right) and the computing speed is considerably less using the p- method. (Figures from GeoFrames internal report 124.081201.huinno.snAps.ir).

3 EXAMPLES

Selected examples show the potential and limitations of today's fracture mechanics modeling. The only code commercially applied on a frequent basis is Fracod2D. The credits include research projects for the Swedish nuclear waste deposition, including the Äspö Pillar Stability Experiment (Shen et al., 2004), analysis of time-dependent behavior of underground constructions in granitic rock in the context of the European DECOVALEX program (Backers et al., 2006a; Shen and Rinne, 2007), and shaft design (Stephansson et al., 2003).

3.1 *Shear Fracture Evolution*

Napier and Backers (2006) analyzed the Punch-Through Shear with Confining Pressure (PTS/CP) experiment using a boundary element approach. Instead of assuming some a priory specific growth sites for cracks, or implementing a general stress based crack initiation criterion, the code considers the problem region to be covered by a random mesh of potential crack segments (Napier and Peirce, 1995; Napier and Malan, 1997). This mirrors the granular structure of the rock material and was considered sensible for micro structural analysis. Crack growth was modeled following a slip- weakening and tension- softening law. Input parameters to the code were mostly estimated from laboratory data.

The PTS/CP geometry and test execution is depicted in Figure 2, additional information may be found in Backers (2005). The PTS/CP experiment is designed to determine the Mode II fracture toughness. The confining pressure, i.e. the normal stress on the propagating fracture, can be applied independently from the shear load. The numerical campaign aimed at mirroring the micro structural breakdown process with increasing confining stress (Figure 3).

The broad conclusions of the study of the behavior of the Punch- Through Shear with Confining Pressure experiment are that qualitative agreement, in terms of both fracture patterns and load- response behavior, can be obtained between the laboratory experiments and the numerical simulation of these tests using a random mesh assembly of displacement discontinuity crack elements. However, significant differences are observed in the shape and magnitude of the overall load- deformation response curves. The numerical experiments also suggest that the relative extent of fracture initiation in tension decreases progressively as the platen displacement is increased when the confining stress is 30 MPa. However, this trend is not displayed when the confining stress is increased to 50 MPa. This corresponds approximately to the apparent suppression of tensile wing cracks in the physical experiments (c.f. Figure 2).

The study reveals as well that it is not straightforward to infer equivalent macro-failure constitutive properties from micro-failure constitutive specifications. This, in turn, suggests that further investigation is required to define an appropriate framework that enables engineering 'strength' properties of rock mass structures to be inferred consistently at different length scales.

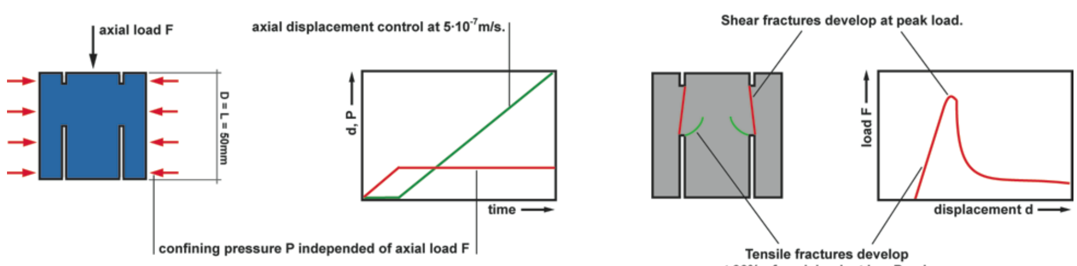

Figure 2. Set-up, geometry and loading of the Punch-Through Shear with Confining Pressure experiment for determination of K_{IIC}. (A) Sample geometry, loading configuration and dimension. (B) After application of confining pressure P an axial load is applied at constant displacement. (C) Typical fracture pattern. First a wing fracture is initiated (but at low P only), subsequently at peak load the shear fracture is connecting the notches. (D) The load F vs. axial displacement d plot shows an increase of load and a clear peak.

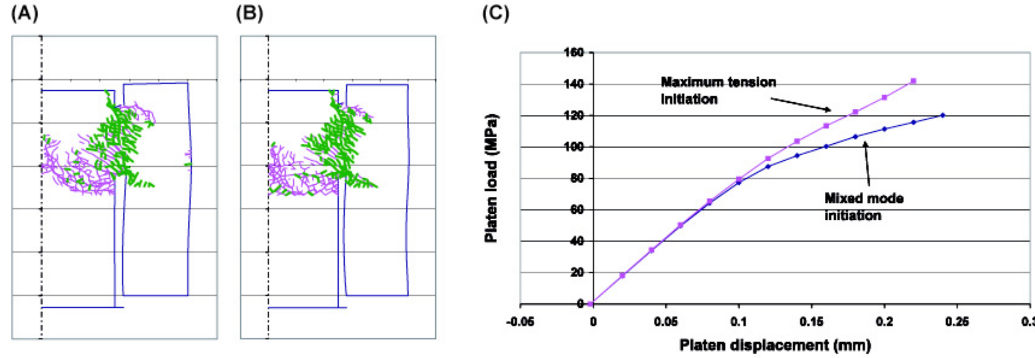

Figure 3. The fracturing development of the PTS/CP experiment was modeled using a BEM approach. The fracture pattern development with different confining pressures (A) of 30MPa and (B) 50MPa could be modeled qualitatively well (platen displacement 0.28 mm; Voronoi tessellation with internal triangulation; green lines depict shear mode cracks, pink lines designate tensile cracks; displacements are magnified by a factor of 10). It was shown that a ‚tension-only‘ criterion is not mirroring the fracturing behavior correctly. (C) shows the effect of failure initiation rule on the simulated load-displacement response.

The main highlight of the study by Napier and Backers (op. cit.) is to emphasize the role of micro-level shear fracturing as a building block for the formation of a macro-shear band structure such as the shear zone that is observed in the PTS/CP test. The numerical experiments reveal that the assumption of fundamental failure rules such as tension-only failure at a micro-scale will not, in general, support the formation and growth of complex shear structures.

3.2 *Uniaxial Compression Experiment - Class II behavior*

Wawersik (1968) identified two fundamental modes of stress–strain behavior in uniaxial compression: Class I and Class II. A Class I complete stress–strain curve monotonically increases in axial strain; a Class II complete stress–strain curve does not monotonically increase in axial strain. In Class II at the peak stress and some of the following post-peak stress levels, the rock specimen contains more strain energy than is required to continue the failure process, and so energy has to be withdrawn.

A uniaxial compression test was simulated with Fracod2D (Fig. 4 and 5). The parameters were taken from several physical tests on saturated Ävrö granite (Bäckstrom et al., 2008). The main parameters used in the Fracod2D model for simulating the breakdown process under uniaxial compression are the fracture toughness and the crack length. They define the level of stress at which the crack starts to propagate.

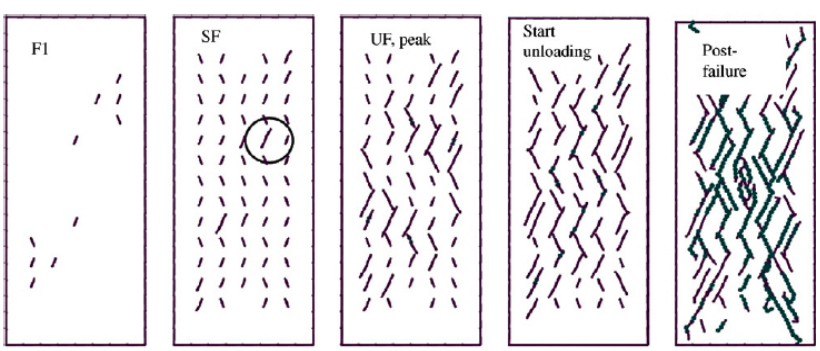

Figure 4. The simulated fracture pattern at fracture initiation (FI), stable fracture propagation (SF), unstable fracture propagation (UF), unloading at the peak stress, and the continuation of cracking.

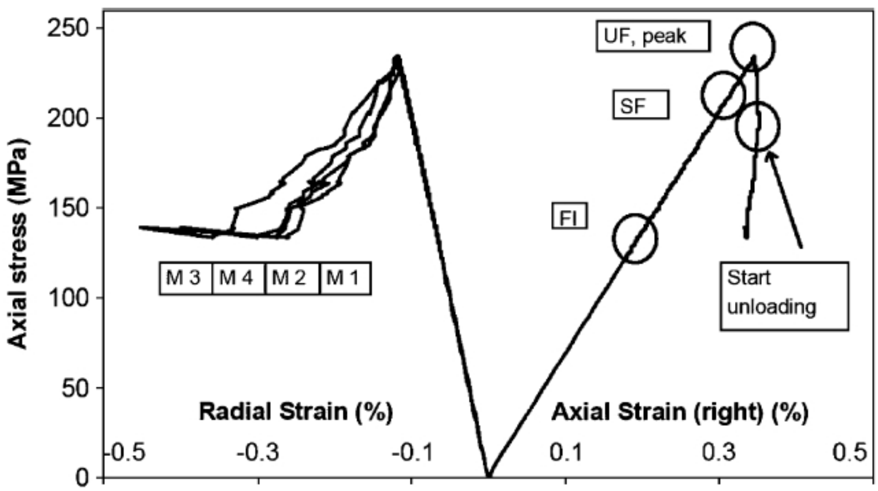

Figure 5. Simulated stress–strain curve illustrating the failure behavior via both radial and axial strains. The situation represented by FI, SF, UF and start unload can be seen in Fig. 5.

Initiation of cracks of given length is defined by the above-mentioned Mohr-Coulomb parameters. The initial crack length is determined from a numerical sensitivity analysis under these conditions with varying crack lengths. The initial crack length was set to 3.125mm in this simulation. The fracture initiation level was set to a stress level of 121MPa, which is the crack initiation strength from physical tests (i.e. about 50% of the UCS and hence the Mohr-Coulomb parameters had to be reduced accordingly).

The reproduction of the behavior of Ävrö granite indicates that shear failure seems to be the dominating failure mode even under simulated uniaxial loading. A sensitivity analysis conducted in connection with this study indicated that the post-peak behavior of the rock is strongly affected by the loading configuration, material properties, etc. The unstable fracturing process of Class II behavior may also cease when a propagating fracture reaches another.

The axial strain was applied in the pre-peak region in small steps equivalent to about a 1MPa increase in the elastic region. After reaching the peak strength, a reversal of the axial strain was applied to be able to simulate the Class II behavior when unstable fracture propagation started. As seen from the results of the simulation, the stable fracture propagation starts at a stress level of 225MPa and, at peak strength for the rock (232MPa), unstable fracturing ensued. To simulate Class II behavior, the unloading was started after passing the peak strength. The progressing failure is detected as increasing radial strain, even if the axial strain is kept constant or even reduced.

Although Fracod2D successfully mirrored the Class II behavior of the ultra-brittle Ävrö granite, and most input parameters, which all have a physical meaning and can in principle be measured in physical experiments, were determined on rock specimens, still tuning of the model had to be performed. The parameters to be adjusted were the initial crack length and the crack initiation stress; both are parameters that are linked to the granular structure of the rock material. The used initial crack length to numerically mirror the peak strength was 3.125mm; the average grain size of the material is about 1.3mm and the maximum grain size, which should be rather the dominating measure when considering fracture mechanics principles can be up to a few centimeters.

3.3 *Stress Field Estimation from Borehole Breakouts*

A non-productive gas exploration well north of Berlin was re-opened as in-situ geothermal laboratory. The in-situ stress state was not well defined and therefore identified borehole breakouts at a depth of ~4,100m in a sandstone formation were used to estimate the stress state. Analysis of the well history revealed a casing lift test as the cause for the breakouts. During the test the mud pressure was decreased from formation pressure of 43.5MPa to 3.7MPa. The width of the breakouts is about 145°. The minimum horizontal stress *Sh* = 50MPa was determined by leak off tests.

The breakouts in the sandstone horizon were analyzed using Fracod2D. The applied boundary stresses for the numerical simulation are *Sh* and the mud pressure *Pw* before and after formation of the borehole breakouts. The unknown parameter maximum horizontal stress *SH* is varied yielding a function to the breakout angle. Comparison of the measured breakout angle and the predicted angle from modeling yields an estimate of the major horizontal stress. The results from the numerical models are plotted in Figure 6, predicting a maximum principle stress of *SH* = 95MPa (Backers et al., 2006b). The predictions of the stress field from the structural geological analysis (Moeck et al., 2008) are in good agreement with the results from the numerical modeling. Further, comparing the uniaxial compressive strength of the sandstone (UCS= 110MPa) with the tangential stress from the Kirsch-solution (Barton and Zoback, 1988) yields *SH* = 93MPa, which is in excellent agreement with the numerical predictions.

The back calculation nicely mirrored the semi- analytical results; as both approaches are based on the same assumptions (macroscopic failure criterion and redistributed stresses around a circular opening) this should be expected. However, the numerical approach does not give any further insights to the understanding of borehole breakouts.

4 CONCLUSIONS AND OUTLOOK

Fracture mechanics based numerical simulations are one step closer towards a physical based model of rock. As pointed out in the introduction, fractures in rock may govern the mechanical and hydro-mechanical behavior of rock. Therefore, it is a logical and consequent move to mirror the processes involved in the formation of fractures by means of material properties. Instead of applying bulk properties for rock mass, which always requires proper tuning of any conventional model, a fracture mechanics approach demands mechanical properties for intact rock, which can be determined in the laboratory for readily available rheological models, and properties describing fracture generation and behavior. The latter are all physically based and can mostly be determined in the laboratory or field.

As was shown in the examples outlined before, the propagation of fractures and generation of fracture networks can be simulated in two dimensions already today. The next step towards a comprehensive fracture mechanics solution for rock is the extension to three spatial dimensions. This is not realized convincingly yet. The mathematical constraints for the extension are not well defined and the numerical tools for this task need to be developed. There are some efforts in this direction undertaken currently.

Rock and rock mass display discontinuities, i.e. cracks, fractures, joints or faults, on different scales. The discontinuous nature of rock is reflected by the DIGS code through an imprinted grid, which fails in the given example to reflect the mesoscopic breakdown process in conjunction with the energy demand. Fracod2d uses an equivalent macro-failure constitutive law; hence the granular nature is not addressed and restricts the code to addressing of the fracturing processes in the meter scale.

As discussed by Napier and Backers (2006) and as manifested in the basic theory for the extension of fractures, the longer a fracture, the smaller the energy required for its extension. This implies for rock or rock mass, that in principle the largest fracture influenced by a change of stress (or any other boundary condition lowering the energy demand for fracture extension) will propagate until the energy is consumed or further fracture propagation is geometrical impossible. In return, this implies that new discontinuities will not be created/initiated, but discontinuities readily available will be activated only. How to introduce realistic sets of discontinuities into such numerical models is not sufficiently solved today without exceeding the numerical capabilities, as the dimensions of discontinuities span several orders of magnitudes. Here the available statistical models have to be reviewed and adapted for the needs of fracture mechanics modeling, bearing the chance to be able to realize scale insensitive models. The most promising approach is the XFEM in combination with the mentioned adaptive hp- approaches.

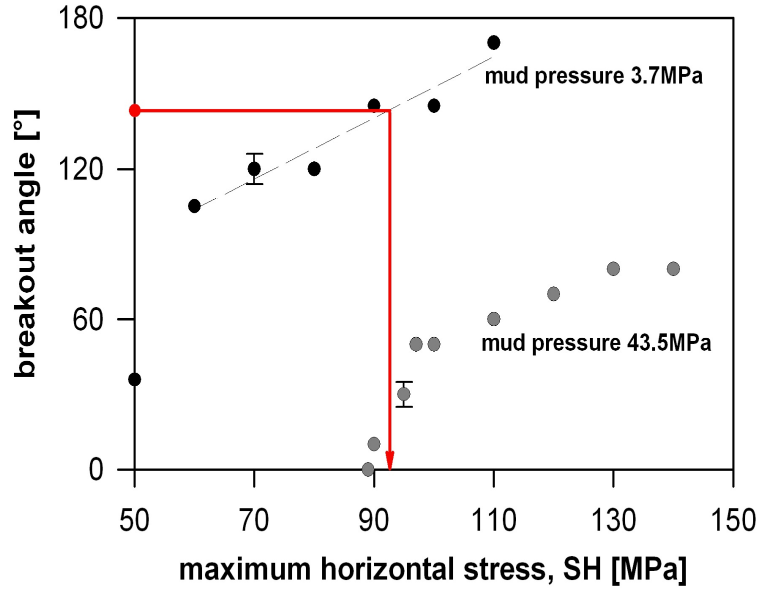

Figure 6. Results from modeling the borehole breakouts at two fluid pressures, i.e. Pw=43.5MPa and Pw=3.7MPa. The breakout angle is known from FMI; from the data SH can be estimated to 95MPa. At high mud pressure only small breakouts could be expected from the analysis.

Based on the combination of readily available models and rock fracture mechanics there is a large potential for different geomechanical applications. In geothermal projects such an approach could help analyze the risk for a thermo-/hydraulic- shortcut or optimize the drilling operations in the reservoir, where underbalanced drilling is favorable in some cases. In the radioactive waste disposal industries, the fracture mechanics approach will help and already has helped understanding the risk for the creation of potential pathways for radionuclide transport and optimize the layout of underground repositories from a long term safety and performance assessment point of view. Slope stability analyses will be able to focus on fracture interaction rather than single discontinuity analysis and hence improve the reliability of the predictions. In the reservoir mechanics applications the factors influencing borehole instabilities or sand production might be identified with such analyses. Hydraulic stimulation campaigns could be analyzed with the aim of an optimized connection of the existing fracture network to the wellbore.

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